Obtaining the necessary concepts in a partial formal context ⋆

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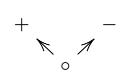
1 Preliminares

In this work, following the notation in [1], $+$ and $-$ will be used for affirmative and negative data, respectively, and missing information will be represented by $\circ$. The set of truth values $\{+,-,\circ\}$. Thus, the extension of formal context is called *partial formal context* and it is a tuple (G,M,I) where G and M are sets, the so-called sets of objects and attributes, respectively and $I\colon G\times M\to\{+,-,\circ\}$ is the incidence relation.

Consider the $\wedge$-semilattice structure, denoted by $\underline{\mathbf{3}}=(\mathbf{3},\preceq)$ as depicted in Figure 1a.



(a) $\wedge$-semilattice $\underline{\mathbf{3}}$

The set of all attributes M considering the three truth-values will be denoted by $\mathbf{3}^M$ and are named consistent sets, and can be endowed with an order relation by extending the order of the truth-values in a pointwise manner. Even though in practice an object will never have and not have an attribute at the same time, it might happen during calculations that such a contradiction arises, therefore the oxymoron, denoted by ¿, will be considered. Hereinafter the set of $\mathbf{3}^M\cup\{\text{¿}\}$ will be denoted by $\dot{\mathbf{3}}^M$, and will be given a lattice structure via the Dedekind-MacNeille completion of $\sqsubseteq$.

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The Galois connection to consider unknown information is $(\ )^{\uparrow}\colon \mathcal{P}(G) \to \dot{\mathbf{3}}^M$ and $(\ )^{\downarrow}\colon \dot{\mathbf{3}}^M \to \mathcal{P}(G)$ defined as

$$X^{\uparrow} = \bigwedge_{g\in X} I_g = \bigwedge_{g\in X} \{{}^{x}/_{I(g,x)} : x \in M\}, \qquad \text{and} \qquad Y^{\downarrow} = \{g \in G \mid I_g \wedge M = M\}.$$

being $I_g = I(g,x)$ for all $x \in M$ and $\bigwedge$ denote the infimum as classically.

A *concept* is a pair $(A,B) \in \mathbf{2}^G \times \dot{\mathbf{3}}^M$ such that $A^{\uparrow} = B$ and $B^{\downarrow} = A$.

2 NextClosure algorithm for partial formal contexts

First, let us assume an arbitrary linear order on the attributes, i.e., $M = \{m_1 < m_2 < \cdots < m_n\}$, and define the following order in $\mathbf{3}$: $\circ < - < +$ note that $<$ is a total order in $\mathbf{3}$. With this idea, a total order on $\dot{\mathbf{3}}^M$ can be defined. Given $A, B \in \dot{\mathbf{3}}^M$, a strict order can be defined by: $A < B$ if $B = \imath$ and $A \neq \imath$ or if there is a m_i such that $A(m_i) < B(m_i)$ and for all $j < i$, one has $A(m_j) = B(m_j)$. Finally, an order relation $\leq$ can be defined by:

$$A \leq B \text{ if and only if } A < B \text{ or } A(i) = B(i) \ \ \forall i \in \{1,\ldots,n\}.$$

Algorithm 1 captures all the concepts. In that algorithm, $\texttt{nextOne}(X)$ is a function that return the smallest set Y such that $X < Y$ and the smallest element b where X has changed, that is, b is the $\mathbf{3}^M$ set such that $b(m_j) = k$ for some j and $b(m_i) = \circ$ for all $i \neq j$ thus $X(m_i) \neq \texttt{nextOne}(X)(m_i) = k$ and each concept is printed just once by using the $\texttt{head}(B,b)$. In addition, Algorithm 1 always ends and computes all the concepts.

Input: A partial formal context $\mathbb{P} = (G, M, I)$
Output: All the formal concepts of the partial formal context
$B := \varepsilon^{\downarrow\uparrow}$
print $\left(B^{\downarrow}, B\right)$
while $B \neq M$ **do**
  $(B,b) := \texttt{nextOne}(B)$
  if $\texttt{head}(B^{\downarrow\uparrow}, b) = \texttt{head}(B,b)$ **then**
    print $\left(B^{\downarrow}, B^{\downarrow\uparrow}\right)$
    $B := B^{\downarrow\uparrow}$
  end
end
print $(\varnothing, \imath)$

Algorithm 1: $\texttt{NextClosure}(\mathbb{P})$

References

1. Rodríguez-Jiménez, J., Cordero, P., Enciso, M., Rudolph, S.: Concept lattices with negative information: A characterization theorem. Information Sciences **369**, 51–62 (2016)