Progress in Formal Context Transforms

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Keywords: Lattice Theory · Concept Lattices · Qualitivative Information

[4] Formal Independence Analysis (FIA) [3] and Formal Equivalence Analysis (FEA) [4] are construction-analogues to FCA in that, first, a basic notion of a formal pair of extent- and intent-analogues are defined, then a result concerning the analysis of the information content of a binary table in terms of those concepts is proposed—notably stating the latticial organization of the set of pairs—and, finally, a result concerning the synthesis of a corresponding data table from a lattice based on pairs built from the join- and meet-irreducibles.

The existence of such analysis-synthesis analogues of the Basic Theorem of FCA [2] allows us to define the concept of a Formal Context Transform (FCT) between formal contexts and several types of lattices built with either upsets and downsets (FCA), or pairs of complementary antichains (FIA), or partitions of the equivalences induced by the polars on the sets of formal objects and attributes (FEA).

In FCA, the notion of a pair of an upset and downset encodes, by analogy, the relation between extent and intent formalized initially by Frege, then by Birkoff [1] and finally by Wille [5]. But in FIA the mathematical object considered are formal *tomoi*, pairs of antichains whose union is a maximal antichain for an order emerging from the formal context considered as a bipartite graph, one of its isomorphic structures. Likewise, in FEA the formal pair of related partitions represent joint, simultaneous resolutions at which formal objects and attributes should be considered for exploration, with monotonously increasing refinement-coarsening orders in each isomorphic lattice.

Since the lattices of these pairs involve different mathematical entities—*viz.* upsets and downsets, anti-chains and equivalences—they impinge differently on the user perceiving the related pairs of isomorphic lattices. We called *formal qualia* the impingement on the mathematical acumen of the use of such pairs to describe informational content in the formal context. In this way we say that FCA provides formal qualia related

to the hierarchical ordering of formal concepts or attributes as induced by the upsets or downsets they belong to as described by the incidence relation. Analogously, FIA provides formal qualia related to the independence relationship among formal objects or attributes, and likewise FEA provides formal qualia related to the undistinguishability of these.

We wonder whether there is any other information content in the formal context. One of the main affordances of FCT is that once we become acquainted and gain understanding of the mathematical abstractions underlying a particular formal analysis, its main formal abstractions also become formal qualia, e.g. formal concepts, formal tomoi, formal partitions, etc. In fact they are very special formal qualia in that they are derived—not primitive—and they include pairs of dual elements. By this we mean that the subjective perception of formal concepts in a concept lattice is quite a different from that of nodes in a lattice: the former has two facades whereby to apprehend them and both are productive when trying to reason about them, e.g. extensions vs. intensions.

Is it reasonable to expect other types of FCT to arise, that is, to extract some other type of information from formal contexts? This depends on our capability to perceive other formal qualia, with the requisite that they be ordered, in the epistemological framework of formal contexts.

However, once we acknowledge that there actually *are* other basic formal qualia, we can be fairly sure that there will be a derived type of quale with a duality and an associated type of lattice evidencing the behaviour of such (qualitative) information whereby we may apprehend yet another angle of formal contexts.

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