Multi-Adjoint and One-Sided Concept Lattices*

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Abstract. Formal Concept Analysis (FCA) is a powerful mathematical tool in order to manage and extract information from databases. There are several frameworks which have been developed in parallel, as one-sided concept lattices and multi-adjoint concept lattices. This work concerns the research recently published in *M. J. Benítez-Caballero, J. Medina, E. Ramírez-Poussa "Characterizing one-sided formal concept analysis by multi-adjoint concept lattices", Mathematics 10 (7): 1020, 2022*; and *M. J. Benítez-Caballero and J. Medina "Characterizing clarify and reduce method by means of left-sided formal concept analysis", Studies in Computational Intelligence 1127:79–85, 2024.*

Formal Concept Analysis (FCA) [14] is a highly effective mathematical tool designed to efficiently manage information and minimize database size. Within FCA, various approaches are employed, including the consideration of fuzzy subsets of attributes and objects. For example, multi-adjoint formal concept analysis [12] offers a generalized framework by utilizing adjoint triples to define concept-forming operators [6–8]. Additionally, the study of (generalized) one-sided formal concept analysis [11], as originally explored by Krajči, involves treating one set - either the set of objects or the set of attributes - as fuzzy and the other as crisp. This paper aims to investigate the relationship between these two theories.

Reducing the size of a database is a critical objective. Therefore, a thorough study of the notion of reduct and an in-depth analysis of different mechanisms for obtaining reducts are imperative.

In [4], we presented the concept of a left-sided adjoint triple, which plays a crucial role in characterizing the (generalized) one-sided formal concept analysis as a specific instance of multi-adjoint formal concept analysis. This enables us to leverage all the principles established within the multi-adjoint structure in the context of the one-sided framework. More specifically, the attribute reduction mechanisms and algorithms developed in the multi-adjoint framework [1]

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can be extended to the one-sided framework. Consequently, certain ideas and findings pertaining to the reduction mechanisms in the multi-adjoint concept lattice framework will be relevant to the one-sided framework. This study was extended in [5] in order to characterize the clarify and reduce method by means of left-sided formal concept analysis.

In future work, we will thoroughly examine property-oriented concept lattices and object-oriented concept lattices [13] to gain a deeper understanding of their relationship. Furthermore, we will actively apply the philosophy of one-sided concept lattice to streamline an information system in Rough Set Theory [2, 3].

References

1. Antoni, L.; Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. Attribute classification and reduct computation in multi-adjoint concept lattices. *IEEE Trans. Fuzzy Syst.* **2021**, *29*, 1121–1132.
2. Benítez-Caballero, M.J.; Medina, J.; Ramírez-Poussa, E. FCA attribute reduction in information systems. In *Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and Foundations*; Medina, J., Ojeda-Aciego, M., Verdegay, J.L., Pelta, D.A., Cabrera, I.P., Bouchon-Meunier, B., Yager, R.R., Eds.; Springer International Publishing: Cham, Switzerland, 2018; pp. 549–561.
3. Benítez-Caballero, M.J.; Medina, J.; Ramírez-Poussa, E.; Ślęzak, D. Bireducts with tolerance relations. *Inf. Sci.* **2018**, *435*, 26–39.
4. Benítez-Caballero, M.J., Medina, J., Ramírez-Poussa, E.: Characterizing one-sided formal concept analysis by multi-adjoint concept lattices. Mathematics **10**(7) (2022). DOI: 10.3390/math10071020,
5. M. J. Benítez-Caballero and J. Medina. Characterizing clarify and reduce method by means of left-sided formal concept analysis. *Studies in Computational Intelligence* **2024**, *1127*, 79–85.
6. Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. A comparative study of adjoint triples. *Fuzzy Sets Syst.* **2013**, *211*, 1–14.
7. Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. Algebraic structure and characterization of adjoint triples. *Fuzzy Sets Syst.* **2021**, *425*, 117–139.
8. Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. Implication operators generating pairs of weak negations and their algebraic structure. *Fuzzy Sets Syst.* **2021**, *405*, 18–39.
9. Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. Attribute reduction in multi-adjoint concept lattices. *Inf. Sci.* **2015**, *294*, 41–56.
10. Cornejo, M.E.; Medina, J.; Ramírez-Poussa, E. Characterizing reducts in multi-adjoint concept lattices. *Inf. Sci.* **2018**, *422*, 364–376.
11. Krajči, S. Cluster based efficient generation of fuzzy concepts. *Neural Netw. World* **2003**, *13*, 521–530.
12. Medina, J.; Ojeda-Aciego, M.; Ruiz-Calviño, J. Formal concept analysis via multi-adjoint concept lattices. *Fuzzy Sets Syst.* **2009**, *160*, 130–144.
13. Medina, J. Multi-adjoint property-oriented and object-oriented concept lattices. *Inf. Sci.* **2012**, *190*, 95–106.
14. Wille, R. Restructuring lattice theory: An approach based on hierarchies of concepts. In *Ordered Sets*; Rival, I., Ed.; Reidel, Dordrecht, Netherlands: 1982; pp. 445–470.