Towards the aggregation of multi-adjoint concept lattices*

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Formal concept analysis allows us to obtain information from relational databases, which are typically represented as object-attribute tables, by formalizing its treatment with the notions of *formal contexts* and *concept lattices*. Aggregation methods [1] are used to link different contexts without modifying the information contained in their concept lattice. One such method relies on the notion of *bond* [2], which is defined as a relation between the set of objects of the first context and the set of attributes of the second context satisfying one condition: if we consider the context obtained with the objects of the first context, attributes of the second context, and the bond as the relation between these sets, the extents of this new context must be extents of the first context and the intents must be intents of the second context.

There are several extensions of the notion of bond to the fuzzy case. In [3] and [4], the authors deal with the L-fuzzy generalization of this notion via complete residuated lattices. More recently, this notion has been defined for the fuzzy case provided by the multi-adjoint framework [5]. A *multi-adjoint bond* is defined not as a relation but as a context with the objects of the first context and attributes of the second one satisfying the condition on its extents and intents mentioned for the classical case. In this paradigm, contexts are associated to a multi-adjoint frame, which means that the properties of the adjoint triples within the frame are of great importance in determining

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whether these multi-adjoint bonds exist. This extension stands out as one of the most flexible and versatile, making it particularly well-suited for modeling real-world problems.

In this first study of this notion, we have focused on two possible multi-adjoint bonds: one characterized by a fuzzy relation that is always the top element of the poset, and another characterized by a fuzzy relation that is always the bottom element of the poset, whenever these elements exist. The first one always results in a multi-adjoint bond, meanwhile there exists sufficient conditions to guarantee the second one also defines a multi-adjoint bond. These conditions assume that the adjoint triples involved have no zero divisors. Moreover, we study how the information is preserved in the aggregated context via their multi-adjoint concept lattices. The two multi-adjoint bonds discussed show respectively how the concept lattices can be aggregated via the cartesian product, binding completely all the original information, and via the horizontal sum, merging independently the information.

References

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