# **(\CEPTS'24**



# **BOOK OF ABSTRACTS**



International Joint Conference

on Conceptual Knowledge Structures

September 10th - 13th, 2024. Cádiz, Spain

Editors: Inma P. Cabrera, Sébastien Ferré, Jesús Medina, Sergei Obiedkov



## Book of abstracts of CONCEPTS 2024

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Edition 1<sup>st</sup> First published: 2024

ISBN: 978-84-09-63916-8 Published and printed by: Universidad de Cádiz (Dept. Matemáticas), Spain.

# Table of Contents

Keynote: Multi-adjoint Formal Concept Analysis.         Theory and Applications         María Eugenia Cornejo Piñero	1
Subdirectly irreducible and semisimple double Boolean algebras Leonard Kwuida, Etienne Romuald Temgoua Alomo	2
The Birkhoff completion of finite lattices Mohammad Abdulla, Johannes Hirth and Gerd Stumme	3
Arrow relations in lattices of integer partitions Asma'a Almazaydeh, Mike Behrisch, Edith Vargas-García, Andreas Wachtel	4
Conceptual knowledge modelling for human–AI teaming in data–frugal industrial environments	5
Handling knowledge over moving object trajectories using Formal Concept Analysis Rouaa Wannous, Salah Eddine Boukhetta, Cécile Vincent	6
FCAvizIR: Exploring relational data set's implications using metrics and topics Lola Musslin, Alexandre Bazin, Marianne Huchard, Pierre Martin, Pascal Poncelet, Vincent Raveneau, Arnaud Sallaberry	7
Realizability of rectangular Euler diagrams Dominik Dürrschnabel, Uta Priss	8
Spanning concept trees: algorithms and interaction Tim Pattison	9
A repository for formal contexts Tom Hanika, Robert Jäschke	10
Keynote: Contextual Attribute Logics Bernhard Ganter	11
Exploring the 3-dimensional variability of websites' user-stories using Triadic Concept Analysis	12

Exploring old arabic remedies with Formal and Relational Concept	
Analysis	13
Conceptual mapping of controversies Claude Draude, Dominik Dürrschnabel, Johannes Hirth, Viktoria Horn, Jonathan Kropf, Jörn Lamla, Gerd Stumme and Markus Uhlmann	14
Towards a generalized modus ponens based on the $\varphi\text{-index}$ of inclusion Carolina Díaz-Montarroso, Nicolás Madrid, Eloísa Ramírez-Poussa	15
Efficiency of fuzzy decision algorithms based on the strength of decision rules Fernando Chacón-Gómez, María Eugenia Cornejo Piñero, Jesús Medina	16
Comparing Relational Concept Analysis and Graph-FCA on their common ground	17
What can FCA-based Boolean matrix factorization do for object-attribute biclustering?	18
Analyzing the relationship between the factorization of formal contexts and independent subcontexts Roberto G. Aragón, Jesús Medina and Eloísa Ramírez-Poussa	19
<b>Keynote:</b> Interactions between the psychology of concepts and conceptual exploration of data	20
Rearrangement of fuzzy formal contexts for reducing cost of algorithms $L \acute{o} pez$ -Rodríguez, Manuel Ojeda-Hernández	21
Enhancing JAAD with knowledge graphs for improved pedestrian crossing predictions	22
Symbolic artificial intelligence for schema therapy using knowledge graphs Madalina Croitoru, Nathalie Blanc, Royce Anders	23
Semantic explorations in factorizing Boolean data via formal concepts $\ . \ .$ Radim Bělohlavék, Martin Trnecka	24

Description lattices of generalised convex hulls	25
Shapley value in classification problems with Triadic Formal Concept Analysis	26
What is the <i>intrinsic</i> dimension of your binary data? – and how to         compute it quickly         Tom Hanika, Tobias Hille	27
Aggregation of fuzzy graphs Francisco Javier Talavera, Carlos Bejines, Sergio Ardanza-Trevijano and Jorge Elorza	28
Bimorphisms and attribute implications in heterogeneous formal contexts L'ubomír Antoni, Peter Eliaš, Ján Guniš, Dominika Kotlárová, Stanislav Krajči, Ondrej Krídlo, Pavol Sokol and L'ubomír Šnajder	29
Discovering the structure of odorants in caenorhabditis elegans using Formal Concept Analysis	30
Exploiting Formal Concept Analysis for data modeling in data lakes Anes Bendimerad, Romain Mathonat, Youcef Remil, Mehdi Kaytoue	31
Document classification via stable graph patterns and conceptual AMR graphs	32
Eric George Parakai, Egor Duayrev, Sergei O. Kuznetsov, Ameaeo Napoli	
Interpretability of formal concepts in word co-occurrence matrices Akihiro Maeda, Takuma Torii and Shohei Hidaka	33
Computing stable extensions of argumentation frameworks using Formal Concept Analysis	34
On the impact of sup-compositions in the resolution of multi-adjoint relation equations	35
Keynote: How to agree to disagree: managing conceptual diversity         using standpoint logic         Sebastian Rudolph	36

Connecting concept lattices with bonds between L-fuzzy formal contexts by external information Ondrej Krídlo, Domingo López-Rodríguez, L'ubomír Antoni, Peter Eliaš, Stanislav Krajči, Manuel Ojeda-Aciego	37
Fuzzy relational Galois connections between fuzzy transitive digraphs Inma P. Cabrera, Pablo Cordero, Emilio Muñoz-Velasco, Manuel Ojeda-Aciego, Bernard de Baets	38
Representation of double Boolean algebra: a summarization and ongoing work Prosenjit Howlader, Churn Jung Liau, and Mohua Banerjee	39
$\begin{array}{llllllllllllllllllllllllllllllllllll$	40
Polyadic Relational Concept Analysis	41
A formalization of multi-label classification in terms of Lattice Theory and Information Theory: concerning datasets Francisco José Valverde-Albacete, Carmen Peláez-Moreno	42

# Program Committee

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# Program of the

# 1<sup>st</sup> International Joint Conference on Conceptual Knowledge Structures

September 10<sup>th</sup> - 13<sup>th</sup>, 2024. Cádiz, Spain

TUESDAY 10 <sup>th</sup>	
Location: Aulario "La Bomba" (Building 1812)	
9:00–9:50	Keynote Speaker - M. Eugenia Cornejo Title: Multi-adjoint Formal Concept Analysis. Theory and Ap- plications Chairperson: Manuel Ojeda-Aciego
9:50-10:50	Session 1. Chairperson: Inma P. Cabrera
	Sub-directly irreducible and semi-simple double Boolean algebras Leonard Kwuida, Etienne Romuald Temgoua Alomo
	The Birkhoff completion of finite lattices Mohammad Abdulla, Johannes Hirth, Gerd Stumme
	Arrow Relations in Lattices of Integer Partitions Asma'a Almazaydeh, Mike Behrisch, Edith Vargas-García, Andreas Wachtel
10:50-11:10	Coffee break
11:10-13:10	Session 2 Chairperson: Gerd Stumme
	Conceptual Knowledge Modelling for Human-AI Teaming in Data-Frugal Industrial Environments Vedran Galetic, Dylan Sheldon, Alistair Nottle
	Handling Knowledge over Moving Object Trajectories using For- mal Concept Analysis Rouaa Wannous, Salah Eddine Boukhetta, Cécile Vincent
	FCAvizIR: Exploring Relational Data Set's Implications using Metrics and Topics Lola Musslin, Alexandre Bazin, Marianne Huchard, Pierre Mar- tin, Pascal Poncelet, Vincent Raveneau, Arnaud Sallaberry
	Realizability of Rectangular Euler Diagrams Dominik Dürrschnabel, Uta Priss
	Spanning Concept Trees: Algorithms and Interaction Tim Pattison
	A Repository for Formal Contexts Tom Hanika, Robert Jäschke
13:10-15:15	Lunch
15:15–16:00	Workshop LBCS Keynote Speaker – Peter Eklund Title: Conceptual Knowledge and large language models Chairperson: L'ubomir Antoni
16:00-17:45	Workshop LBCS
18:00-20:30	Cádiz Tour

WEDNESDAY 11 <sup>th</sup> Location: Aulario "La Bomba" (Building 1812)	
9:00-10:00	<b>Keynote Speaker - Bernhard Ganter</b> Title: Contextual Attribute Logics Chairperson: Jesús Medina
10:00-11:00	Session 3 Chairperson: Dominik Dürrschnabel
	Exploring the 3-Dimensional Variability of Websites' User- Stories using Triadic Concept Analysis Alexandre Bazin, Thomas Georges, Marianne Huchard, Pierre Martin and Chouki Tibermacine
	Exploring Old Arabic Remedies with Formal and Relational Con- cept Analysis Vanessa Fokou, Karim El Haff, Agnès Braud, Xavier Dolques, Florence Le Ber, Véronique Pitchon
	Conceptual Mapping of Controversies Claude Draude, Dominik Dürrschnabel, Johannes Hirth, Vikto- ria Horn, Jonathan Kropf, Jörn Lamla, Gerd Stumme, Markus Uhlmann
11:00-11:20	Coffee break
11:20-12:55	Session 4 Chairperson: Sébastien Ferré
	Towards a generalized modus ponens based on the $\varphi$ -index of inclusion Carolina Díaz-Montarroso, Nicolás Madrid, Eloísa Ramírez-Poussa
	Efficiency of fuzzy decision algorithms based on the strength of decision rules Fernando Chacón-Gómez, María Eugenia Cornejo Piñero, Jesús Medina
	Comparing Relational Concept Analysis and Graph-FCA on their common ground Vanessa Fokou, Peggy Cellier, Xavier Dolques, Sébastien Ferré, Florence Le Ber
	What can FCA-based Boolean matrix factorization do for object- attribute biclustering? Martin Trnecka, Roman Vyjidacek
	(Kw) Analyzing the relationship between the factorization of for- mal contexts and independent subcontexts Roberto G. Aragón, Jesús Medina and Eloísa Ramírez-Poussa
13:00-20:00	Excursion including the lunch

THURSDAY 12 <sup>th</sup>	
Location: Aulario "La Bomba (Building 1812)	
9:00–10:00	<b>Keynote Speaker - Radim Bělohlávek</b> Title: Interactions between the psicology of concepts and con- ceptual exploration of data Chairperson: Pablo Cordero
10:00-11:00	Session 5 Chairperson: Karell Bertet
	Rearrangement of fuzzy formal contexts for reducing cost of al- gorithms Domingo López-Rodríguez, Manuel Ojeda Hernández
	Enhancing JAAD with Knowledge Graphs for Improved Pedes- trian Crossing Predictions Sandra Victor, Gowrishankar Ganesh, Madalina Croitoru
	Symbolic Artificial Intelligence for Schema Therapy using Knowledge Graphs Madalina Croitoru, Nathalie Blanc, Royce Anders
11:00-11:20	Coffee break
11:20-13:20	Session 6 Chairperson: Ondrej Krídlo
	Semantic explorations in Boolean matrix factorization utilizing formal concepts Radim Belohlavek, Martin Trnecka
	Description Lattices of Generalised Convex Hulls Christophe Demko, Karell Bertet, Jean-François Viaud, Cyril Faucher, Damien Mondou
	Shapley value in classification problems with Triadic Formal Concept Analysis Martin Waffo Kemgne, Blaise Bleriot Koguep Njionou, Leonard Kwuida, Dmitry I. Ignatov
	What is the intrinsic dimension of your binary data? and how to compute it quickly Tom Hanika, Tobias Hille
	Aggregation of fuzzy graphs Francisco Javier Talavera, Carlos Bejines, Sergio Ardanza- Trevijano and Jorge Elorza
	Bimorphisms and attribute implications in heterogeneous formal contexts L'ubomír Antoni, Peter Eliaš, Ján Guniš, Dominika Kotlárová, Stanislav Krajči, Ondrej Krídlo, Pavol Sokol, L'ubomír Šnajder
13:20 - 15:00	Lunch

15:00-16:00	<b>Round table.</b> Title: Past, present and future of CONCEPTS. Participants: R. Bělohlavek, M. E. Cornejo, B. Ganter, S. Rudolph
	Chairperson: Jesús Medina
16:00-18:00	Session 7 Chairperson: Sergei Obiedkov
	Discovering the Structure of Odorants in Caenorhabditis elegans using Formal Concept Analysis Emma Reyner-Fuentes, Carmen Peláez-Moreno, Francisco José Valverde-Albacete
	Exploiting Formal Concept Analysis for Data Modeling in Data Lakes Anes Bendimerad, Romain Mathonat, Youcef Remil, Mehdi Kaytoue
	Document Classification via Stable Graph Patterns and Concep- tual AMR Graphs Eric George Parakal, Egor Dudyrev, Sergei O. Kuznetsov, Amedeo Napoli
	(Kw) Interpretability of Formal Concepts in Word Co-occurrence Matrices Akihiro Maeda, Takuma Torii and Shohei Hidaka
	(Kw) Computing Stable Extensions of Argumentation Frame- works using Formal Concept Analysis Sergei Obiedkov and Baris Sertkaya
	(Kw) On the impact of sup-compositions in the resolution of multi-adjoint relation equations David Lobo, Víctor López-Marchante, Jesús Medina
20:30	Gala dinner

FRIDAY 13 <sup>th</sup> Location: Aulario "La Bomba" (Building 1812)	
10:00-11:00	<b>Keynote Speaker - Sebastian Rudolph</b> Title: How to agree to disagree: managing conceptual diversity using standpoint logic. Chairperson: Uta Priss
11:00-11:20	Coffee Break
11:20-13:20	Session 8 Chairperson: Carmen Peláez-Moreno
	(Kw) Connecting concept lattices with bonds between L-fuzzy for- mal contexts by external information Ondrej Krídlo, Domingo López-Rodríguez, Lubomir Antoni, Eliaš, Krajči, Manuel Ojeda-Aciego
	<ul> <li>(Kw) Fuzzy relational Galois connections between fuzzy transi- tive digraphs</li> <li>Inma P. Cabrera, Pablo Cordero, Emilio Muñoz-Velasco, Manuel Ojeda-Aciego, Bernard de Baets</li> </ul>
	(Kw) Representation of Double Boolean Algebra: A Summariza- tion and Ongoing Work Prosenjit Howlader, Churn Jung Liau, and Mohua Banerjee
	<ul> <li>(Kw) Concept-forming operators of multi-adjoint concept lattices with hedges</li> <li>M. Eugenia Cornejo, Jesús Medina and Francisco José Ocaña</li> </ul>
	(Kw) Polyadic Relational Concept Analysis Alexandre Bazin, Jessie Galasso and Giacomo Kahn
	(Kw) A Formalization of Multi-Label Classification in terms of Lattice Theory and Information Theory: Concerning Datasets Francisco Valverde-Albacete, Carmen Peláez-Moreno
13:20-15:00	Closing Lunch Session

Key works (Kw) will have 10 minutes for the presentation of the paper, plus 5 minute for questions.

The rest of contributions will have 15 minutes for the presentation, plus 5 minutes for questions.

Keynote speech:

# Multi-adjoint Formal Concept Analysis. Theory and Applications

#### María Eugenia Cornejo Piñero

Universidad de Cádiz, Spain

**Abstract:** Due to the inherent imperfection in real databases the fuzzy extension of formal concept analysis (FCA) was a necessity. Multi-adjoint formal concept analysis, introduced by Medina and Ojeda-Aciego, arose as a fuzzy framework of formal concept analysis focused on introducing a general approach capable of conveniently embed the different fuzzy extensions of concept lattices given in the literature, such as the original framework given by Burusco and Fuentes-González, and the so-called approaches given by Belohlavek, Pollandt, Georgescu, Popescu and Krajci. For instance, a general algebraic structure was considered in which the operators could be non-commutative and nonassociative, allowing a more flexible modeling of the examples/dataset. Another interesting feature is that it is possible to consider different adjoint triples or pairs with respect to a given triple of posets, which allows to offer the possibility of considering different degrees of preference on the attributes/objects of a database. These preference degrees can be interpreted as the values of a membership function modeling a preference on the attributes and/or objects, following the semantics proposed by Zadeh, in which the values represent the intensity of preference in favor of a specific attribute/object. Attribute reduction and size reduction of the concept lattice, and the computation of reducts are significant challenges in this theory, which is fundamental for detecting redundant information and selecting the most relevant variables in the (real) dataset. Different approaches have been introduced with remarkable results. Attribute implications theory is another fundamental challenge in FCA, where recent advances have also been obtained and they have been applied to real cases with good results. This presentation provides a wide view on multi-adjoint formal concept analysis including theoretical definitions, properties, and important tasks where this theory has successfully been applied, such as the commented above attribute reduction and attribute implications theory. In addition, different applications will be shown.

#### Session 1

# Subdirectly irreducible and semisimple double Boolean algebras

Leonard Kwuida<sup>1</sup> and Etienne Romuald Temgoua Alomo<sup>2</sup>

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<sup>2</sup> University of Yaounde I, École Normale Supérieure de Yaoundé Department of Mathematics, P.O. Box 47, Yaoundé, Cameroon e-mail: retemgoua@gmail.com

**Abstract:** Double Boolean algebras are algebras  $\underline{D} = (D, \sqcap, \sqcup, \neg, \lrcorner, \bot, \top)$  of type (2, 2, 1, 1, 0, 0) introduced by Rudolf Wille to capture the equational theory of protoconcept algebras. A famous theorem of Birkhoff says that any variety is determined by its subdirectly irreducible members. In this work we give a construction that leads to a concrete embedding of double Boolean algebras into the protoconcept algebra. We characterize subdirectly irreducible, simple and semisimple double Boolean algebras.

### The Birkhoff completion of finite lattices

Mohammad Abdulla<sup>1, 2</sup>, Johannes Hirth<sup>1,2</sup> and Gerd Stumme <sup>1,2</sup>

<sup>1</sup> Knowledge & Data Engineering Group, University of Kassel, Germany

 $^2$  Interdisciplinary Research Center for Information System Design, University of Kassel, Germany

**Abstract:** We introduce the Birkhoff completion as the smallest distributive lattice in which a given finite lattice can be embedded as semi-lattice. We discuss its relationship to implicational theories, in particular to R. Wille's simply-implicational theories and truncated distributive lattices. By an example, we show how the Birkhoff completion can be used as a tool for ordinal data science.

### Arrow relations in lattices of integer partitions

Asma'a Almazaydeha<sup>1</sup>, Mike Behrischb<sup>2</sup>, Edith Vargas-García<sup>3</sup> and Andreas Wachtel<sup>3</sup>

 $^1$  Department of Mathematics, Tafila Technical University, PO Box 179, 66110, Tafila, Jordan

 $^2$ Institute of Discrete Mathematics and Geometry, TU<br/> Wien, Wiedner Hauptstr. 8–10, 1040, Vienna, Austria

 $^3$  Department of Mathematics, ITAM, Río Hondo 1, Ciudad de México, CP 01080, Mexico

**Abstract:** We give a complete characterisation of the single and double arrow relations of the standard context  $\mathbb{K}(\mathcal{L}_n)$  of the lattice  $\mathcal{L}_n$  of partitions of any positive integer n under the dominance order, thereby addressing an open question of Ganter, 2020/2022.

**DOI:** https://doi.org/10.1016/j.ijar.2024.109244

#### Session 2

# Conceptual knowledge modelling for human-AI teaming in data-frugal industrial environments

#### Vedran Galetic,<sup>1</sup> Dylan Sheldon<sup>1</sup> and Alistair Nottle<sup>1</sup>

<sup>1</sup> Airbus Central R&T, Bristol, United Kingdom. e-mail:{vedran.galetic,dylan.sheldon,alistair.nottle}@airbus.com

**Abstract:** When AI interacts with humans in complex environments, such as aerospace manufacturing, safety of operation is of paramount importance. Trustworthiness of AI needs to be ensured through, among other things, explainability of its behaviour and rationale, which is typically a challenge for current deep neural network-based systems. We tackle the knowledge comprehensibility aspect of intrinsic explainability by suggesting a concept-level environment awareness model combining various complementary knowledge sources — statistical learning using dedicated property detectors through publicly available software, and crowd-sourced common-sense knowledge graphs. Our approach also addresses the issue of data-frugal learning, typical for environments with highly specific purpose-built artefacts. We adopt Gärdenfors's Conceptual Spaces as a cognitively-motivated knowledge representation framework and apply our typicality quantification model in a use case on interpretable classification of manufacturing artefacts.

### Handling knowledge over moving object trajectories using Formal Concept Analysis

#### Rouaa Wannous<sup>1</sup>, Salah Eddine Boukhetta<sup>1</sup>, and Cécile Vincent<sup>1</sup>

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**Abstract:** For us, the term semantic trajectories of moving objects corresponds to a sequence of spatio-temporal points with associated semantic information. Spatio-temporal points are directly generated by sensors that capture the position of moving object in time.

In this paper, we work on a marine mammal trajectories case study. In fact, we consider seals' trajectories in order to understand their behavior as groups and identify their activities together at the same time in the same place. We define mobile object's activities in form of rules given by the domain expert. To accomplish knowledge over trajectories, we use the platform GALACTIC, which is a new platform based on Formal Concept Analysis (FCA) for calculating a concept lattice from heterogeneous and complex data. Data in GALACTIC are described by predicates according to their types. Here, we will use interval sequences plugins to analyse seals' trajectories, where interval sequences are represented by a set of time intervals in which a seal perform an activity in a geographical zone. Finally, the results show activities of group of seals simultaneously.

### FCAvizIR: Exploring relational data set's implications using metrics and topics

Lola Musslin<sup>1</sup>, Alexandre Bazin<sup>1</sup>, Marianne Huchard<sup>1</sup>, Pierre Martin<sup>2, 3</sup>, Pascal Poncelet<sup>1</sup>, Vincent Raveneau<sup>1</sup> and Arnaud Sallaberry<sup>1, 4</sup>

<sup>1</sup>LIRMM, Univ. Montpellier, CNRS, Montpellier, France.

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<sup>3</sup>AIDA, Univ. Montpellier, CIRAD, Montpellier, France.

<sup>4</sup>AMIS, Univ. Paul-Valéry Montpellier 3, Montpellier, France.

Abstract: Implication is a core notion of Formal Concept Analysis and its extensions. It provides information about the regularities present in the data. When one considers a relational data set of real-size, implications are numerous and their formulation, which combines primitive and relational attributes computed using Relational Concept Analysis framework, is complex. For an expert wishing to answer a question based on such a corpus of implications, having a smart exploration strategy is crucial. In this paper, we propose a visual approach, implemented in a web platform named FCAvizIR, for leveraging such corpus. Comprised of three interactive and coordinated views and a toolbox, FCAvizIR has been designed to explore corpora of implication rules following Schneiderman's famous mantra "overview first, zoom and filter, then details on demand". It enables metrics filtering, e.g. fixing a minimum and a maximum support value, and the multiple selection of relations and attributes in the premise and in the conclusion to identify the corresponding subset of implications presented as a list and Euler diagrams. An example of exploration is presented using an excerpt of Knomana to analyze plant-based extracts for controlling pest.

#### Realizability of rectangular Euler diagrams

#### Dominik Dürrschnabel,<sup>1,2</sup> and Uta Priss<sup>3</sup>

<sup>1</sup> Knowledge & Data Engineering Group, University of Kassel, Kassel, Germany

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**Abstract:** Euler diagrams are a tool for the graphical representation of set relations. Due to their simple way of visualizing elements in the sets by geometric containment, they are easily readable by an inexperienced reader. Euler diagrams where the sets are visualized as aligned rectangles are of special interest. In this work, we link the existence of such rectangular Euler diagrams to the order dimension of an associated order relation. For this, we consider Euler diagrams in one and two dimensions. In the one-dimensional case, this correspondence provides us with a polynomial-time algorithm to compute the Euler diagrams, while the two-dimensional case is linked to an NP-complete problem which we approach with an exponential-time algorithm.

### Spanning concept trees: algorithms and interaction

#### **Tim Pattison**

Defence Science & Technology Group West Ave, Edinburgh South Australia 5111 e-mail: tim.pattison@defence.gov.au

Abstract: The Concept Tree is a means of browsing the concept lattice of a formal context using readily-available tools for the interactive visualisation of trees. While respecting the lattice order, the directed Concept Tree includes arcs between concepts which are not mutual covers, and hence is not a spanning tree of the lattice digraph. Such arcs misrepresent the structure of the digraph. This paper surveys and augments options for constructing a Spanning Concept Tree, including: as an auxiliary data structure of existing Formal Concept Analysis algorithms; transitive reduction of the partial order amongst previously-enumerated concepts to produce the lattice digraph, from which a spanning tree is then derived; a novel algorithm which exploits the Reverse Lectic Order (RLO) of intents to construct a spanning tree from the set of formal concepts; and a novel algorithm which identifies and remediates only those arcs in the Concept Tree which are not arcs in the lattice digraph. Both novel algorithms exploit the property that a post-order traversal of the Concept Tree returns the concepts in RLO. They produce Spanning Concept Trees in which the parent of each concept is its first cover in the RLO, and whose children, like those in the Concept Tree, are in RLO. However, some concepts in the Spanning Concept Tree may depart from their canonical positions in the Concept Tree. The implications of this departure for interactive exploration vice algorithmic traversal - of the Spanning Concept Tree are also explored.

### A repository for formal contexts

#### Tom Hanika<sup>1</sup> and Robert Jäschke<sup>2</sup>

- <sup>1</sup> University of Hildesheim. Hildesheim, Germany e-mail: tom.hanika@uni-hildesheim.de
- <sup>2</sup> Berlin School for Library and Information Science Humboldt-Universität zu Berlin. e-mail: : robert.jaeschke@hu-berlin.de

**Abstract:** Data is always at the center of the theoretical development and investigation of the applicability of Formal Concept Analysis. It is therefore not surprising that a large number of de facto standard data sets are repeatedly used in scientific articles and software tools. However, this distribution of data sets poses a problem for the sustainable further development of the research field. There is a lack of a centralized location that holds and describes FCA datasets and links already known analysis results. This article analyses the current state of the dissemination of FCA datasets, presents the requirements for a central FCA repository and highlights the challenges for this.

#### Keynote speech:

### **Contextual Attribute Logics**

#### **Bernhard Ganter**

TU Dresden, Germany

Abstract: Mathematical formal logic is a comprehensive and profound theory that is valuable and significant also for Formal Concept Analysis. However, it is not to be understood as "concept logic" in the philosophical sense. Its language and technical terms are geared towards its main field of application, mathematics. That is why Rudolf Wille began early on to formulate a logic of formal concepts under the name "Contextual Logic". A first building block for this is Contextual Attribute Logic. However, the initial publication on this topic remained largely unnoticed because it did not show much that was mathematically new. In fact, a reader could quite rightly say that this attribute logic is merely reformulated Propositional Logic (and more generally that Contextual Logic can be translated into Predicate Logic). However, this very argument was also put forward for years to Formal Concept Analysis itself. The fact that a reformulation, guided by an interpretative goal, and the associated restructuring can release completely new forces, was only understood when Formal Concept Analysis was taken up by the academic world in thousands of scientific publications. A suitably coordinated logical language is part of such a restructuring. In our talk, we will present Contextual Attribute Logic again, going a few steps further than the previously published contributions. As always, our goal is to provide a framework that is suitable for holding together the divergent developments within Formal Concept Analysis and contributes to a more unified theory.

#### Session 3

# Exploring the 3-dimensional variability of websites' user-stories using Triadic Concept Analysis

Alexandre Bazin<sup>1</sup>, Thomas Georges<sup>1, 2</sup>, Marianne Huchard<sup>1</sup>, Pierre Martin<sup>3</sup> and Chouki Tibermacine<sup>1</sup>

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Abstract: Configurable software systems and families of similar software systems are increasingly being considered by industry to provide software tailored to each customer's needs. Their development requires managing software variability, i.e. commonalities, differences and constraints. A primary step is properly analyzing the variability of software, which can be done at various levels, from specification to deployment. In this paper, we focus on the software variability expressed through user-stories, viz. short formatted sentences indicating which user role can perform which action at the specification level. At this level, variability is usually analyzed in a two dimension view, i.e. software described by features, and considering the roles apart. The novelty of this work is to model the three dimensions of the variability (i.e. software, roles, features) and explore it using Triadic Concept Analysis (TCA), an extension of Formal Concept Analysis. The variability exploration is based on the extraction of 3- dimensional implication rules. The adopted methodology is applied to a case study made of 65 commercial web sites in four domains, i.e. manga, martial arts sports equipment, board games including trading cards, and video-games. This work highlights the diversity of information provided by such methodology to draw directions for the development of a new product or for building software variability models.

DOI: https://doi.org/10.1016/j.ijar.2024.109248

# Exploring old arabic remedies with Formal and Relational Concept Analysis

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**Abstract:** Exploring old pharamacopeia is a promising way to find active ingredients that can be useful to design new drugs. Nevertheless, studying these texts is a laborious task for biologists. Therefore, an interdisciplinary project was undertaken: texts have been annotated to extract relevant information and represent it within a graph database. Formal Concept Analysis (FCA) and Relational Concept Analysis (RCA) have then been used to explore this database, in order to answer questions regarding remedies and their ingredients. This paper presents the data and some results obtained with FCA and RCA. It highlights the suitability of these approaches to explore these data and answer biologist needs.

#### **Conceptual mapping of controversies**

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**Abstract:** With our work, we contribute towards a qualitative analysis of the discourse on controversies in online news media. For this, we employ Formal Concept Analysis and the economics of conventions to derive conceptual controversy maps. In our experiments, we analyze two maps from different news journals with methods from ordinal data science. We show how these methods can be used to assess the diversity, complexity and potential bias of controversies. In addition to that, we discuss how the conceptual diagrams can be used to navigate between news articles.

#### Session 4

# Towards a generalized modus ponens based on the $\varphi$ -index of inclusion

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**Abstract:** This paper proposes a generalized modus ponens and a generalized modus tollens based on the  $\varphi$ -index of inclusion. Moreover, we analyze the properties of generalized modus ponens and generalized modus tollens according to the axioms proposed by Baldwin and Pilsworth.

### Efficiency of fuzzy decision algorithms based on the strength of decision rules

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**Abstract:** Characterizing data sets through logic rules is a fundamental task for modeling knowledge systems. Mathematical tools, such as Rough Set Theory (RST) and Formal Concept Analysis (FCA), offer mechanisms for automatically obtaining these logic rules. In this paper, we will analyse a special kind of set of logic rules in fuzzy rough set theory called decision algorithm. In particular, a definition of efficiency of a decision algorithm is introduced taking into account a new fuzzy relevance indicator based on the strength of the decision rules.

### Comparing Relational Concept Analysis and Graph-FCA on their common ground

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**Abstract:** Relational Concept Analysis (RCA) and Graph-FCA (GCA) have been defined as Formal Concepts Analysis (FCA) extensions for processing relational data and knowledge graphs respectively. Nevertheless, whereas their purposes seem similar, and their results may appear to be identical, the data modeling and the definition and subsumption of concepts are different. In this paper, we compare these two approaches on a common basis, considering only unary and binary relations for GCA and the existential quantifier for RCA. We focus on examples showing the similarities and dissimilarities between the two methods, and highlighting how cycles are processed differently by RCA and GCA.

## What can FCA-based Boolean matrix factorization do for object-attribute biclustering?

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**Abstract:** We introduce preliminary results from an ongoing study on FCA-based Boolean Matrix Factorization (BMF) and its application in object-attribute biclusters enumeration. Namely, we show that standard methods may overlook some object-attribute biclusters, and we propose a novel factor extension approach that can add some of the overlooked biclusters. The presented method computes biclusters employing logic similar to that of standard methods. Additionally, we outline the utilization of a well-known technique used in BMF and present topics for future research.

# Analyzing the relationship between the factorization of formal contexts and independent subcontexts

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**Abstract:** This is a keywork of the paper: R. G. Aragón, J. Medina, and E. Ramírez-Poussa. *Factorizing formal contexts from closures of necessity operators*. Comp. Appl. Math, 43(124), 2024.

DOI: https://doi.org/10.1007/s40314-024-02590-0

#### Keynote speech:

## Interactions between the psychology of concepts and conceptual exploration of data

#### Radim Bělohlavék

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**Abstract:** Both the psychology of concepts and conceptual exploration of data have developed a rich body of knowledge (repertoire of theories and tools). Naturally, these two areas have different aims. We argue that despite their focus on different phenomena, proper interactions between the psychology of concepts and conceptual exploration of data may benefit both areas. For this purpose, we provide examples of recent work involving human categorization, typicality, similarity, and factor analysis.

#### Session 5

# Rearrangement of fuzzy formal contexts for reducing cost of algorithms

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**Abstract:** The influence of attribute ordering on the runtime efficiency of FCA algorithms has long been a subject of conjecture, yet no prior published work has directly addressed this issue. This paper proposes a novel approach, introducing criteria for ranking attribute importance within an *L*-fuzzy formal context. The primary objective is to reduce the runtime of concept lattice construction algorithms by strategically reordering attributes based on these criteria.

# Enhancing JAAD with knowledge graphs for improved pedestrian crossing predictions

#### Sandra Victor<sup>1</sup>, Gowrishankar Ganesh<sup>1</sup> and Madalina Croitoru<sup>1</sup>

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**Abstract:** In this paper we place ourselves in the broad context of autonomous driving and, more precisely, in the context of road crossing decisions by pedestrians. Machine learning models built by the existing work need as training data datasets constructed by semi-automatic annotation of video images. JAAD (Joint Attention in Autonomous Driving) is a popular behavior annotated dataset in this regard. Our contribution is to provide a knowledge graph based proposal of additional features that are important but missing in JAAD dataset in the context of road crossing decisions by pedestrians.

# Symbolic artificial intelligence for schema therapy using knowledge graphs

#### Madalina Croitoru<sup>1,2</sup>, Nathalie Blanc<sup>2</sup>, and Royce Anders<sup>2</sup>

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**Abstract:** In this paper we place ourselves in the broader context of Artificial Intelligence (AI) powered mental health apps and, more precisely, in the context of semi-automated schema based therapy. Our contribution is two fold. First we provide the first rule based ontology in the literature for Schema Therapy and demonstrate how it can be practically used by mental health professionals in supporting their clients. Second we pave the way for explainable, symbolic based AI approaches in mental health that are yet to be investigated alongside the more prominent approaches (currently in fashion) based on large language models (LLMs).

#### Session 6

# Semantic explorations in factorizing Boolean data via formal concepts

#### Radim Bělohlavék<sup>1</sup> and Martin Trnecka<sup>1</sup>

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**Abstract:** We use now available psychological data involving human concepts, objects covered by these concepts, and binary attributes describing the objects to explore selected semantic aspects of Boolean matrix factorization. Our basic perspective derives from the intuitive requirement that the factors computed from data should represent nat- ural categories latently present in the data. This idea is examined for factorization algorithms that utilize formal concepts to build factors. We provide several experimental observations which imply that the inspected factorization methods deliver semantically sound factors that resemble significant human concepts of the examined domains.

**DOI:** https://doi.org/10.1016/j.ijar.2024.109247

# Description lattices of generalised convex hulls

# Christophe Demko<sup>1</sup>, Karell Bertet<sup>1</sup>, Jean-François Viaud <sup>1</sup>, Cyril Faucher<sup>1</sup> and Damien Mondou<sup>1</sup>

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**Abstract:** In this article, we present a new approach to tackle lattice generation for complex and heterogeneous data using the concept of convexity. This is a work that we have already carried out, albeit intuitively, where we proposed the NextPriorityConcept algorithm for generating a meet-semilattice of concepts based on suitable descriptions and strategies. Now, we revisit the essential properties of our description spaces using a stronger formalism based on the properties of closure operators.

DOI: https://doi.org/10.1016/j.ijar.2024.109269

### Shapley value in classification problems with Triadic Formal Concept Analysis

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**Abstract:** The JSM-method is a supervised classification method, widely used in machine learning. It has recently been used in Triadic Concept Analysis to classify objects. In this paper, we show how Shapley value of a cooperative game with transferable utilities, can be used to give the importance or individual contribution of each attribute-condition pair of a particular object, for its classification to a particular class.

### What is the intrinsic dimension of your binary data? – and how to compute it quickly

#### Tom Hanika<sup>1</sup>, Tobias Hille<sup>2,3</sup>

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**Abstract:** Dimensionality is an important aspect for analyzing and understanding (high-dimensional) data. In their 2006 ICDM paper Tatti et al. answered the question for a (interpretable) dimension of binary data tables by introducing a normalized fractal dimension. In the present work we revisit their results and contrast them with a concept based notion of intrinsic dimension (ID) recently introduced for geometric data sets. To do this, we present a novel approximation technique for this ID that is based on computing concepts only up to a certain support value. We demonstrate and evaluate our approximation using all still available datasets from Tatti et al., which have between 469 and 41271 extrinsic dimensions. (Source code and more figures are available at https://codeberg. org/thille/bd-gid).

### Aggregation of fuzzy graphs

#### Francisco Javier Talavera<sup>1, 2</sup>, Carlos Bejines<sup>2</sup>, Sergio Ardanza-Trevijano<sup>1,2</sup> and Jorge Elorza<sup>1, 2</sup>

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<sup>3</sup> Univesidad de Málaga, Andalucía Tech, 29071, Málaga, Spain.

**Abstract:** Abstract: Our study is centered on the aggregation of fuzzy graphs, looking for conditions under which the aggregation process yields another fuzzy graph. We conduct an in-depth analysis of the preservation of several important properties and structures inherent to fuzzy graphs, like paths, cycles, or bridges. In addition we obtain appropriate criteria for when the aggregation of complete fuzzy graphs is again a complete fuzzy graph.

DOI: https://doi.org/10.1016/j.ijar.2024.109243

# Bimorphisms and attribute implications in heterogeneous formal contexts

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Abstract: Formal concept analysis is a powerful mathematical framework based on mathematical logic and lattice theory for analyzing object-attribute relational systems. Over the decades, Formal concept analysis has evolved from its theoretical foundations into a versatile methodology applied across various disciplines. A heterogeneous formal context provides a feasible generalization of a formal context, enabling diverse truthdegrees of objects, attributes, and fuzzy relations. In our paper, we present extended theoretical results on heterogeneous formal contexts, including bimorphisms, Galois connections, and heterogeneous attribute implications. We recall the basic notions and properties of the heterogeneous formal context and its concept lattice. Moreover, we present extended results on bimorphisms and Galois connections in a heterogeneous formal context, including a self-contained proof of the main result. We include examples of introduced notions in heterogeneous formal contexts and two-valued logic. We propose the extension of attribute implications for heterogeneous formal contexts and explore their validity. By embracing heterogeneity in Formal concept analysis, we enrich its extended theoretical foundations and pave the way for innovative applications across diverse domains, including personal data protection and cybersecurity.

**DOI:** https://doi.org/10.1016/j.ijar.2024.109245

#### Session 7

# Discovering the structure of odorants in caenorhabditis elegans using Formal Concept Analysis

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**Abstract:** Embodied and Embedded cognition posits that animals act by operating a sensory-motor loop over an environment through the concourse of their nervous systems. A notable challenge in studying the nervous system lies in the absence of a model of its sensorium, the mechanism whereby an organism perceives its surroundings. This paper addresses the inquiry into the perception mechanisms of Caenorhabditis elegans as a model organism. To this end, we have delimited the perceptual domain to 23 distinct odorants, for which we have the corresponding neural activity data from the 11 pairs of sensory neurons. This paper elucidates how the integration of various FCA-related methodologies—specifically, K-FCA max-plus and min-plus analyses, as well as cosine distance calculations—allows for the obtention of a representation of the perceptual framework of C. elegans. Additionally, we propose avenues for further refinement, including the consideration of the idempotent singular value decomposition within the FCA framework to improve said perceptual representations.

# Exploiting Formal Concept Analysis for data modeling in data lakes

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Abstract: Data lakes are widely used to store extensive and heterogeneous datasets for advanced analytics. However, the unstructured nature of data in these repositories introduces complexities in exploiting them and extracting meaningful insights. This motivates the need of exploring efficient approaches for consolidating data lakes and deriving a common and unified schema. This paper introduces a practical data visualization and analysis approach rooted in Formal Concept Analysis (FCA) to systematically clean, organize, and design data structures within a data lake. We explore diverse data structures stored in our data lake at Infologic, including InfluxDB measurements and Elasticsearch indexes, aiming to derive conventions for a more accessible data model. Leveraging FCA, we represent data structures as objects, analyze the concept lattice, and present two strategies—top-down and bottom-up—to unify these structures and establish a common schema. Our methodology yields significant results, enabling the identification of common concepts in the data structures, such as "resources" along with their underlying shared fields (timestamp, type, usedRatio, etc.). Moreover, the number of distinct data structure field names is reduced by 54% (from 190 to 88) in the studied subset of our data lake. We achieve a complete coverage of 80% of data structures with only 34 distinct field names, a significant improvement from the initial 121 field names that were needed to reach such coverage. The paper provides insights into the Infologic ecosystem, problem formulation, exploration strategies, and presents both qualitative and quantitative results. The source code and datasets of this work are made available: https://zenodo.org/records/10589722

# Document classification via stable graph patterns and conceptual AMR graphs

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Abstract: This paper proposes an approach and an associated system based on pattern structures, aimed at the classification of documents represented as graphs. The representation of documents relies on Abstract Meaning Representation (AMR) document graphs. Given a set of AMR document graphs, the system learns characteristic graph patterns, that can be reused by an aggregate rule classifier to predict the class of a document. The selection of the most stable graph patterns is based on the gSOFIA algorithm and the  $\Delta$ -stability measure. In the experiments, two document datasets are considered for validating the approach. The first includes documents belonging to 10 different newsgroups and the second contains sports news articles belonging to 5 topical areas. The results in terms of the macro-averaged F1 scores, are quite satisfactory and show that the approach is well-founded and useful.

# Interpretability of formal concepts in word co-occurrence matrices

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**Abstract:** This study addresses the interpretability of word representations through an investigation of a word co-occurrence matrix. Employing Formal Concept Analysis, we aimed to reveal an underlying structure that is consistent with human interpretation. We unveil the emergence of hierarchical and geometrical structures within word vectors. We show that the identified formal concepts align with interpretable categories.

Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (ACL 2024): https://aclanthology.org/2024.findings-acl.278

## Computing stable extensions of argumentation frameworks using Formal Concept Analysis

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**Abstract:** We propose an approach based on Formal Concept Analysis (FCA) for computing stable extensions of Abstract Argumentation Frameworks (AFs). To this purpose, we represent an AF as a formal context in which stable extensions of the AF are closed sets called concept intents. We make use of algorithms developed in FCA for computing concept intents in order to compute stable extensions of AFs. Experimental results show that, on AFs with a high density of the attack relation, our algorithms perform significantly better than the existing approaches. The algorithms can be modified to compute other types of extensions, in particular, preferred extensions.

**DOI:** https://doi.org/10.1007/978-3-031-43619-2\_13

# On the impact of sup-compositions in the resolution of multi-adjoint relation equations

David Lobo<sup>1</sup>, Víctor López-Marchante<sup>1</sup>, Jesús Medina<sup>1</sup>

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**Abstract:** Keywork of paper On the impact of sup-compositions in the resolution of multi-adjoint relation equations Mathematical Methods in the Applied Sciences, 2023, 46, 15581-15598".

DOI: https://doi.org/10.1002/mma.9414

#### Keynote speech:

### How to agree to disagree: managing conceptual diversity using standpoint logic

#### Sebastian Rudolph

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**Abstract:** The importance of taking individual, potentially conflicting perspectives into account when dealing with knowledge has been widely recognised. Many existing ontology management approaches fully merge knowledge perspectives, which may require weakening in order to maintain consistency; others represent the distinct views in an entirely detached way. This talk presents an alternative, referred to as Standpoint Logic, a simple, yet versatile multi-modal logic "add-on" for existing KR languages intended for the integrated representation of domain knowledge relative to diverse standpoints, which can be hierarchically organised, combined, and put in relation with each other. Starting from the generic framework of First-Order Standpoint Logic (FOSL), we first present the fragment of so-called sentential formulas, for which we provide a polytime translation into the standpoint-free version. This result yields decidability and favourable complexities for several decidable fragments of first-order logic, including the very expressive description logic SROIQbs underlying the OWL 2 DL ontology language. By virtue of this, existing highly optimised OWL reasoners can be used to provide practical reasoning support for ontology languages extended by standpoint modelling. Shifting our focus to tractable lightweight formalisms of enhanced scalability, we present Standpoint EL+, a standpoint extension of the popular description logic EL. Satisfiability in this logic can be checked in polynomial time thanks to a satisfiability-checking deduction calculus that allows for a implementation by means of elementary logic programming.

#### Session S8

## Connecting concept lattices with bonds between *L*-fuzzy formal contexts by external information

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**Abstract:** We have delved into the interpretation of two novel methods for selecting appropriate bonds between L-fuzzy formal contexts, based on the use of the rigorous and benevolent concept-forming operators. The strategy presented therein for the construction of the rigorous and the benevolent bonds is knowledge-driven: the presence of external information about the strength of the connection about the attribute sets, given by operator  $p: A_1 \times A_2 \to L$ , induces both types of bond. Therefore, these new methods overcome the difficulty of bond interpretation by taking advantage of the incorporation of knowledge into the problem. In that paper, we formally verified that the bonds built using this strategy produce results coherent with the piece of external information used.

DOI: https://doi.org/10.1016/j.ins.2023.119498

# Fuzzy relational Galois connections between fuzzy transitive digraphs

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**Abstract:** We present a fuzzy version of the notion of relational Galois connection between fuzzy transitive directed graphs (fuzzy T-digraphs) on the specific setting in which the underlying algebra of truth values is a complete Heyting algebra. The components of such fuzzy Galois connection are fuzzy relations satisfying certain reasonable properties expressed in terms of the so-called full powering. Moreover, we provide a necessary and sufficient condition under which it is possible to construct a right adjoint for a given fuzzy relation between a fuzzy T-digraph and an unstructured set.

DOI: https://doi.org/10.1016/j.fss.2022.12.012

# Representation of double Boolean algebra: a summarization and ongoing work

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**Abstract:** In the line of, Wille (2000), Balbiani (2012) and Vormbrock (2005), in this abstract, we present results on the topological representation of dBa based on Howlader and Banerjee (2023), and our ongoing work on its algebraic representation.

**DOI:** https://doi.org/10.1007/s00012-023-00811-x

# Concept-forming operators of multi-adjoint concept lattices with hedges

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**Abstract:** Formal Concept Analysis is an appealing research topic both from a theoretical and applied perspective. Since its introduction in the eighties, different fuzzy generalizations have been proposed in order to deal with the imprecise and vague information contained in databases. Specifically, multi-adjoint concept lattices arise as a general and flexible approach capable of conveniently embed different fuzzy extensions of concept lattices mentioned above. Attribute classification, size lattice reduction, reduct computation and attribute implications are important tasks which have been successfully addressed in the multi-adjoint paradigm.

This work studies the inclusion of truth-stressing hedges in multi-adjoint concept lattices. The formal definition of concept-forming operators of multi-adjoint concept lattices with hedges and its properties are presented. In addition, an isomorphism between a multi-adjoint concept lattice and the concept lattice enriched with truth-stressing hedges is obtained from the new operators.

DOI: https://doi.org/10.1016/j.fss.2023.108854

### **Polyadic Relational Concept Analysis**

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Abstract: Formal concept analysis is a mathematical framework based on lattice theory that aims at representing the information contained in binary object-attribute datasets (called formal contexts) in the form of a lattice of so-called formal concepts. Since its introduction, it has been extended to more complex types of data. In this paper, we are interested in two of those extensions: relational concept analysis and polyadic concept analysis that allow to process, respectively, relational data and n-ary relations. We present a framework for polyadic relational concept analysis that extends relational concept analysis to relational datasets that are made of n-ary relations. We show its basic properties and that it is a valid extension of relational concept analysis.

**DOI:** https://doi.org/10.1016/j.ijar.2023.109067

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# A formalization of multi-label classification in terms of Lattice Theory and Information Theory: concerning datasets

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**Abstract:** Multi-label classification (MLC) is a relatively recently-formalized task in Machine Learning. We have proven that this establishes a metaphor of MLC as an information communication channel, using information triangles to quantify information and concept lattices to qualify it.

DOI: https://doi.org/10.3390/math12020346

# **Author Index**

Abdulla, Mohammad, 3 Almazaydeha, Asma'a, 4 Anders, Royce, 23 Antoni, L'ubomír, 29, 37 Aragón, Roberto G., 19 Ardanza-Trevijano, Sergio, 28

Bělohlavék, Radim, 20, 24 Banerjee, Mohua, 39 Bazin, Alexandre, 7, 12, 41 Behrischb, Mike, 4 Bejines, Carlos, 28 Bendimerad, Anes, 31 Bertet, Karell, 25 Blanc, Nathalie, 23 Boukhetta, Salah Eddine, 6 Braud, Agnès, 13

Cabrera, Inma P., 38 Cellier, Peggy, 17 Chacón-Gómez, Fernando, 16 Cordero, Pablo, 38 Cornejo Piñero, María Eugenia, 1, 16, 40 Croitoru, Madalina, 22, 23

Díaz-Montarroso, Carolina, 15 de Baets, Bernard, 38 Demko, Christophe, 25 Dolques, Xavier, 13, 17 Draude, Claude, 14 Dudyrev, Egor, 32 Dürrschnabel, Dominik, 8, 14

El Haff, Karim, 13 Eliaš, Peter, 29, 37 Elorza, Jorge, 28

Faucher, Cyril, 25

Ferré, Sébastien, 17 Fokou, Vanessa, 13, 17 Galasso, Jessie, 41 Galetic, Vedran, 5 Ganesh, Gowrishankar, 22 Ganter, Bernhard, 11 Georges, Thomas, 12 Guniš, Ján, 29 Hanika, Tom, 10, 27 Hidaka, Shohei, 33 Hille, Tobias, 27 Hirth. Johannes. 3. 14 Horn, Viktoria, 14 Howlader, Prosenjit, 39 Huchard, Marianne, 7, 12 Ignatov, Dmitry I., 26 Jäschke, Robert, 10 Kahn, Giacomo, 41 Kaytoue, Mehdi, 31 Kemgne, Martin Waffo, 26 Koguep Njionou, Blaise Bleriot, 26 Kotlárová, Dominika, 29 Krídlo, Ondrej, 29, 37 Krajči, Stanislav, 29, 37 Kropf, Jonathan, 14 Kuznetsov, Sergei O., 32 Kwuida, Leonard, 2, 26

López-Marchante, Víctor, 35 López-Rodríguez, Domingo, 21, 37 Lamla, Jörn, 14 Le Ber, Florence, 13, 17 Liau, Churn Jung, 39 Lobo, David, 35 Madrid, Nicolás, 15 Maeda, Akihiro, 33 Martin, Pierre, 7, 12 Mathonat, Romain, 31 Medina, Jesús, 16, 19, 35, 40 Mondou, Damien, 25 Musslin, Lola, 7 Muñoz-Velasco, Emilio, 38

Napoli, Amedeo, 32 Nottle, Alistair, 5

Obiedkov, Sergei, 34 Ocaña, Francisco José, 40 Ojeda-Aciego, Manuel, 37, 38 Ojeda-Hernández, Manuel, 21

Parakal, Eric George, 32 Pattison, Tim, 9 Peláez-Moreno, Carmen, 30, 42 Pitchon,Véronique, 13 Poncelet, Pascal, 7 Priss, Uta, 8

Ramírez-Poussa, Eloísa, 15, 19 Raveneau, Vincent, 7 Remil, Youcef, 31 Reyner-Fuentes, Emma , 30 Rudolph, Sebastian, 36

Sallaberry, Arnaud, 7 Sertkaya, Baris, 34 Sheldon, Dylan, 5 Snajder, L'ubomír, 29 Sokol, Pavol, 29 Stumme, Gerd, 3, 14

Talavera, Francisco Javier, 28 Temgoua Alomo, Etienne Romuald, 2 Tibermacine, Chouki, 12 Torii, Takuma, 33 Trnecka, Martin, 18, 24

Uhlmann, Markus, 14

Valverde-Albacete, Francisco José, 30, 42 Vargas-García, Edith, 4 Viaud, Jean-François, 25 Victor, Sandra, 22 Vincent, Cécile, 6 Vyjidacek, Roman, 18

Wachtel, Andreas, 4 Wannous, Rouaa, 6